

Solutions to Homework Number Six
Due Monday November 14, 2016

1) A copper (pure copper) bar and eutectoid steel (assume the steel is bainite), bar "sitting on a table in the lab" are cut into several pieces. Your supervisor is confused about the behavior of pure copper (which melts at 1060C) and eutectoid steel (the eutectoid temperature is 727C) because of the following observations.

- If I place the steel in a 700C saturated furnace for two hours and then slam it into a water bath; it gets weaker and softer.
- If I place the steel in a 1050C, saturated furnace for two hours and then slam it into a water bath; it gets harder and stronger
- If I place the steel in a 1050C, saturated furnace for two hours and then turn the furnace off and remove the steel when the furnace is at room temperature; it gets weaker and softer.
- If I place the copper in a 700C saturated furnace for two hours and then slam it into a water bath; it gets weaker and softer.
- If I place the copper in a 1050C, furnace for two hours and then slam it into a water bath; it gets even weaker and softer than the copper which was subjected to a similar heat treatment but at 700C.
- If I melt copper, cool to 700C and wait for it to solidify and then slam it into a water bath it is harder and stronger than if I melt copper, cool to 1050C, wait for it to solidify and slam it into a water bath.
- In fact the copper which was melted and then solidified at 700C may be stronger and harder than that which was simply heated to 700C. But, the copper that was melted and then solidified at 1050C may be softer and weaker than the copper that was simply heated to 1050C.

Prepare a report explaining this. Use processing-structure-processing relationships as the basis for your report.

The key to answering this question is two fold.

- First, one needs to determine what structure will form as a result of the processing.
- Second, one needs to relate that structure to strength and hardness, based on the relative number of barriers to dislocation motion.

Do not try to combine these two steps into one.

Some of these processes will result in a phase change, or sequence of phase changes others will not. The eutectoid steel will not become austenite unless it is heated above 727C. Pure copper will not undergo a phase change unless it is melted.

<u>Processes which Result in a Phase Change</u>	<u>Processes which Do Not Result in a Phase Change</u>
<ul style="list-style-type: none"> • If I place the steel in a 1050C, saturated furnace for two hours and then slam it into a water bath • If I place the steel in a 1050C, saturated furnace for two hours and then turn the furnace off and remove the steel when the furnace is at room temperature; • If I melt copper, cool to 700C and wait for it to solidify and then slam it into a water bath • I melt copper, cool to 1050C, wait for it to solidify and slam it into a water bath. 	<ul style="list-style-type: none"> • If I place the steel in a 700C saturated furnace for two hours and then slam it into a water bath; • If I place the copper in a 700C saturated furnace for two hours and then slam it into a water bath; • If I place the copper in a 1050C, furnace for two hours and then slam it into a water bath

Results of Processing where no Phase Change Occurs

When exposed to elevated temperature the underbonded atoms which make up interfaces will move and rebond. This reduces the number of interfaces in the material. In single phase alloys the removal of interfaces results in grain growth (the elimination of grain boundaries). In two phase alloys the removal of interfaces results in coarsening (the elimination of phase boundaries). Since interfaces are barriers to dislocation motion, if the number of interfaces is reduced, then dislocation motion becomes easier and the strength is reduced.

When steel is heated to 700C and held for two hours coarsening occurs. The size of the second phase particles increases as the underbonded atoms associated with the interface move and rebond, thus reducing energy. Thus, the number of phase boundaries per unit area, which are barriers to dislocation motion is reduced, which would make the material weaker and softer.

Copper does not change its crystal structure at high temperature. Therefore, if copper is heated to 700C or to 1050C, grain growth will occur. The underbonded atoms associated with the interface move and rebond, thus reducing energy. Thus, the number of phase boundaries per unit area, which are barriers to dislocation motion is reduced, which would make the material weaker and softer. Because diffusion (the process by which atoms move) increases exponentially with temperature, the increase in grain size, and the corresponding reduction in the number of barriers to dislocation motion is more pronounced.

Results of Processing where Phase Changes Occur

When the steel is heated to 1050C it forms austenite the high temperature form of iron. When austenite is cooled quickly to room temperature it forms martensite. Martensite is a single phase material, the crystal structure of which does not allow for dislocation motion. Thus it is very strong, hard, and brittle.

When solidifying molten metal one must examine the size of the grains which form. This will be determined by the nucleation rate and the growth rate. Recall that the nucleation rate is effected by the rate of diffusion, which increases with temperature, and the number of atoms required to form a

nucleus, which decreases with temperature. When molten copper is cooled to 700C the nucleation rate is high and the growth rate is low. This means a fine structure will form and there will be many barriers to dislocation motion. When molten copper is cooled to 1050C the nucleation rate will be low but the growth rate will be high. This means a coarse structure will form and there will be few barriers to dislocation motion. Therefore melting copper, cool to 700C and waiting for it to solidify and then slamming it into a water bath will result in a finer structure, than if one melts copper, cools to 1050C, waits for it to solidify and slams it into a water bath. So it is likely that the copper which was ,melted, cooled to 700C and allowed to to solidify and then slammed into a water bath is harder and stronger than the copper, which was melted cooled to 1050C, allowed to solidify and slammed into a water bath.

This is also the reason that the copper which was melted and then solidified at 700C may be stronger and harder than that which was simply heated to 700C. The copper which was will form a fine structure, where the copper that was simply heated to 700C will undergo grain growth forming a coarser structure. As the coarser structure has fewer barriers to dislocation motion, it will be softer and weaker.

The copper that was melted and then solidified at 1050C may be softer and weaker than the copper that was simply heated to 1050C, because both will be coarse structures. The sample that was heated to 1050C undergoes extensive grain growth. The copper which was melted and solidified at 1050C forms as a very coarse structure.

When the steel is heated to 1050C it forms austenite the high temperature form of iron. Cooled by simply turning the furnace off and waiting is the slowest possible cooling rate. This means the temperature at which austenite decomposes will be very high and the time at that temperature very long. This is equivalent to holding at 700C for a day or two. Very coarse pearlite will form. The critical nucleus size is large, the nucleation rate is very slow. This means very few large nuclei will form. But when they do form they will grow quickly forming a very coarse structure. Coarse structures have fewer barriers to dislocation motion, so this structure would be softer and weaker than the starting material (tempered martensite).

2) Prepare a technical memorandum describing the results of the TTT Diagram Experiment

The key content is detailed below.

Introduction

Only include background and the presentation of the hypothesis. The hypothesis, or hypotheses, will need to include the following. 1) when and why coarsening is expected to occur, 2) what products are expected from the decomposition of austenite (bainite, martensite, or pearlite) and why they will occur in certain temperature ranges. 3) when and why tempered martensite will form. 4) Explanation of the predicted relative hardnesses.

Methodology

The material, all 11 steels must be listed.

The procedure section of the methodology should be written to the intelligent person standard. It should be 4 sentences at most.

Results

Make sure your description of the predicted and observed microstructures are included.

Discussion

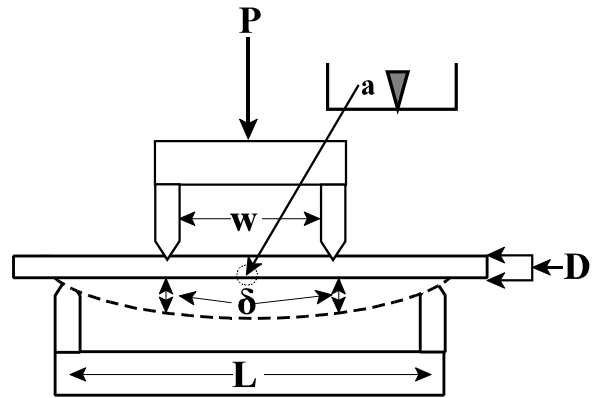
There needs to be a specific references to the results when comparing them to your hypothesis. This includes hardness values and specific microstructures. All 3 result sheets need to be referenced.

3) To determine the mechanical properties of Silicon Nitride a 4 point bend test was performed. In this test

- a 2cm (2r) diameter rod
- with surface cracks 25 microns (a) in length

was placed in a four point bend apparatus such that the

- lower supports were 50cm (l) apart
- and the upper load points were 25 cm (w) apart.
- The rod failed under a load of 38kN (P)
- with a deflection of 3.5mm (δ).



Determine the following assuming that fibers of Silicon Nitride are to be used in a polymer reinforced composite and loaded in pure tension.

- The maximum load that a fiber with a 2.5 square mm cross sectional area could support if the surface cracks were no greater than 5 microns.
- The cross sectional area required to support a 150kN load if the surface cracks were no greater than 15 microns.
- The maximum allowable surface crack length if a fiber with a cross-sectional area of 50 sq mm is to support a load of 25kN.

The testing condition/experimental parameters are listed below.

- Lower Arm Spacing: 50cm (l)
- Upper Arm Spacing: 25cm (w)
- Radius: 1cm (r)
- Crack Size in Rod: 25 microns (a)
- Breaking Load: 38kN (P)
- Deflection at Break: 3.5mm (δ)

The bending stress in the rod undergoing 4 point bend is:

$$\begin{aligned}\sigma &= \frac{P(l-w)}{\pi r^3} \\ &= \frac{(38 \times 10^3 \text{ N})(50 \times 10^{-2} \text{ m} - 25 \times 10^{-2} \text{ m})}{\pi(1.0 \times 10^{-2} \text{ m})^3} \\ &= 3.02 \times 10^9 \text{ Pa} = 3.02 \text{ GPa}\end{aligned}$$

The fracture toughness of the material is:

$$\begin{aligned}
 K_{IC} &= Y\sigma\sqrt{\pi a} \\
 &= (1)(3.02 \times 10^9 \text{ Pa})\sqrt{\pi(25 \times 10^{-6} \text{ m})} \\
 &= 26.8 \times 10^6 \text{ Pa}\sqrt{\text{m}}
 \end{aligned}$$

The fiber is loaded in pure tension as shown. Thus the breaking stress must be determined for each crack length.

The breaking strength of a brittle material with a crack in it can be determined from the fracture toughness. So if the crack length is 5 microns, the breaking strength is determined as follows.

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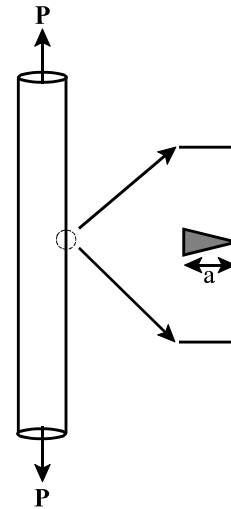
$$\begin{aligned}
 K_{IC} &= Y\sigma\sqrt{\pi a} \\
 \sigma &= \frac{K_{IC}}{Y\sqrt{\pi a}} \\
 &= \frac{26.8 \times 10^6 \text{ Pa}\sqrt{\text{m}}}{(1)\sqrt{\pi(5 \times 10^{-6} \text{ m})}} \\
 &= 6.8 \times 10^9 \text{ Pa}
 \end{aligned}$$

So, based on the above calculations the breaking strength of a fiber with surface cracks of 5 microns is 6.8×10^9 Pa.

Thus, using the definition of stress, we can calculate the maximum load that a fiber with a 2.5 square mm cross sectional area could support.

$$\begin{aligned}
 \sigma &\equiv \frac{F}{A_0} \\
 F &= \sigma A_0 \\
 &= (6.8 \times 10^9 \text{ Pa})(2.5 \times 10^{-6} \text{ m}^2) \\
 &= 17 \times 10^3 \text{ N}
 \end{aligned}$$

Thus, the maximum load that a fiber with a 2.5 square mm cross sectional area could support if the surface cracks were no greater than 5 microns is 17×10^3 N.



Fiber with Surface Cracks in Pure Tension

The fracture toughness of the material is: $26.8 \times 10^6 \text{ Pa}\sqrt{m}$

The fiber is loaded in pure tension as shown. Thus the breaking stress must be determined for each crack length.

The breaking strength of a brittle material with a crack in it can be determined from the fracture toughness. So if the crack length is 15 microns, the breaking strength is determined as follows.

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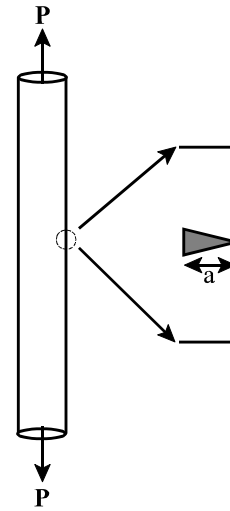
$$\begin{aligned}
 K_{IC} &= Y\sigma\sqrt{\pi a} \\
 \sigma &= \frac{K_{IC}}{Y\sqrt{\pi a}} \\
 &= \frac{26.8 \times 10^6 \text{ Pa}\sqrt{m}}{(1)\sqrt{\pi(15 \times 10^{-6} \text{ m})}} \\
 &= 3.9 \times 10^9 \text{ Pa}
 \end{aligned}$$

If the crack length were 15 microns the breaking stress would be, $3.9 \times 10^9 \text{ Pa}$.

Thus, using the definition of stress, we can calculate the cross sectional area required to support a 150kN load if the surface cracks were no greater than 15 microns.

$$\begin{aligned}
 \sigma &\equiv \frac{F}{A_0} \\
 A_0 &= \frac{F}{\sigma} \\
 &= \frac{150 \times 10^3 \text{ N}}{3.9 \times 10^9 \text{ Pa}} \\
 &= 38 \times 10^{-6} \text{ m}^2
 \end{aligned}$$

Thus, the cross sectional area required to support a 150kN load if the surface cracks were no greater than 15 microns is $38 \times 10^{-6} \text{ m}^2$



Fiber with Surface Cracks in Pure Tension

The maximum allowable crack length can also be calculated using the definition of fracture toughness. The breaking stress required for fiber with a 50 sq mm cross-sectional area which must support a load of 25kN is,

$$\begin{aligned}\sigma &= \frac{F}{A_0} \\ &= \frac{25 \times 10^3 N}{(50 \times 10^{-6} m^2)} \\ &= 500 \times 10^6 Pa\end{aligned}$$

Using the definition of fracture toughness, one can determine the maximum allowable crack length for a required breaking strength

$$\begin{aligned}K_{IC} &= Y\sigma\sqrt{\pi a} \\ \sqrt{\pi a} &= \frac{K_{IC}}{Y\sigma} \\ a &= \frac{1}{\pi} \left(\frac{K_{IC}}{Y\sigma} \right)^2 \\ &= \frac{1}{\pi} \left(\frac{26.8 \times 10^6 Pa\sqrt{m}}{(1)(500 \times 10^6 Pa)} \right)^2 \\ &= 910 \times 10^{-6} m\end{aligned}$$

Thus the maximum crack length is $910 \times 10^{-6} m$.

4) Compare fracture toughness and strength. Use the results of Problem 3 as part of your comparison.

Strength is the stress at which something happens. For example,

- the yield strength is the stress at which yielding occurs,
- the ultimate tensile strength is the maximum stress on the engineering stress-strain curve and coincides with the on-set of necking.

Strength is used to determine maximum loading conditions for ductile materials.

Fracture toughness is the key material property to determine if brittle materials will fail. It is based on two factors

- the applied stress σ
- the crack length a

Fracture Toughness K_{IC} is defined as

$$K_{IC} = Y\sigma\sqrt{\pi a}$$

where Y is a geometrical factor (1 unless otherwise noted)

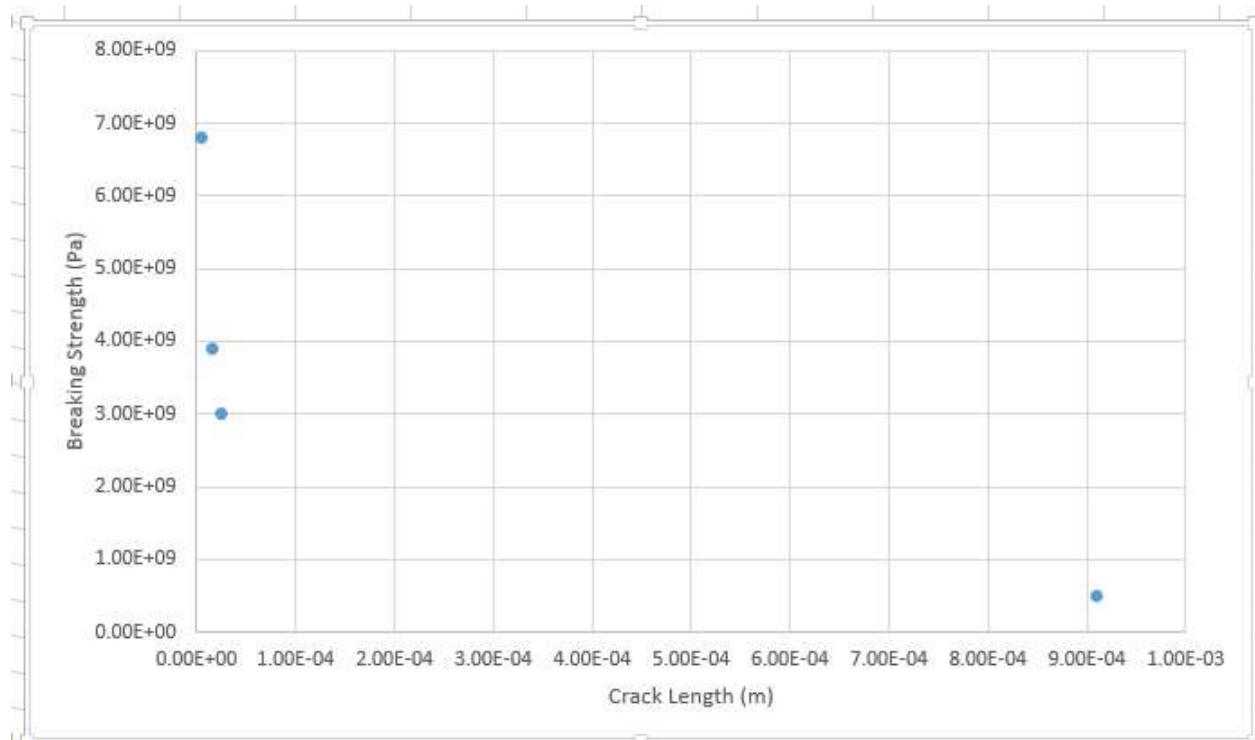
Thus the breaking strength of a brittle material is dependent on the crack length

$$\sigma = \frac{K_{IC}}{Y\sqrt{\pi a}}$$

The table below shows the data from Problem 3.

Crack length (m)	Breaking Strength (Pa)
25×10^{-6}	3000×10^6
5×10^{-6}	6800×10^6
15×10^{-6}	3900×10^6
910×10^{-6}	500×10^6

This means that as the crack size increases the breaking strength decreases.



5) Compare the use of the “homologous temperature” and “solidification temperature” tables. Use Aluminum which has a melting temperature of 660C as the basis for the examples used to justify your comparison.

An example calculation for determining the temperature in Celsius which corresponds to 85% of the melting temperature of aluminum (melting temperature - 660C) is shown below.

$$\frac{T}{T_M} = 85\%$$

$$T = (0.85)(660C + 273) - 273$$

$$= 520C$$

$\frac{T}{T_M}$	The homologous temperature (fraction of melting point) is considered	$\frac{T}{T_M}$	The solidification temperature is considered
75-100%	high	90-100%	high
50-75%	intermediate	75-90%	intermediate
<50%	low	<75%	low

High: T > 427C	High: >567C
Intermediate T: 194C-427C	Intermediate T: 427C-567C
Low T: < 194C	Low T: < 427C

